A Study of Laminar-flow Heat Transfer in Tubes

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The general problem of heat transfer to fluids in laminar flow in tubes is discussed, a new procedure for the measurement of local laminar-flow heat transfer coefficients is described, and an empirical equation is presented for the correlation of data for local heat transfer rates to liquids flowing upward in laminar flow in vertical tubes under conditions of constant heat flux at the tube wall.

The study of laminar-flow heat transfer in tubes has continued both experimentally and analytically ever since the pioneer work of Graetz (*1*) in 1883. Most such work has been directed toward describing this phenomenon through the use of arbitrarily defined heat transfer coefficients which apply to a finite length of tube and include entrance effects. The purpose of this paper is to describe a new experimental technique for the measurement of local heat transfer rates in laminar flow and to present a preliminary correlation of such data for the heating of liquids in tubes.

Heat transfer to fluids in fully developed turbulent flow has long been adequately handled by means of heat transfer coefficients, primarily because of the existence of a laminar boundary layer or film at the tube wall. In the core of the fluid the temperature varies but little over a cross section because of the high degree of turbulence. Only in the laminar film does the temperature rise sharply to the wall temperature. Thus for the case of turbulent flow the bulk temperature of the fluid is essentially the temperature of the turbulent core. Heat transferred to the core must pass through the laminar boundary layer. Since mixing does not occur in this film, the mechanism of heat transfer through it is conduction, which occurs at a rate given by the basic equation

$$dq = -k\left(\frac{dt}{ds}\right) dA \qquad (1)$$

where dq is the differential rate of heat transfer across the differential area dA, k is the thermal conductivity, and dt/ds is the temperature gradient in the direction of heat transfer, perpendicular to the plane of A.

For the case of turbulent flow, the temperature gradient in the laminar film is approximately $\Delta t/\delta$, where Δt is the difference between the wall temperature and the bulk temperature of the fluid and δ is the film thickness. Thus

$$-\frac{dt}{ds} = \frac{dt}{dr} = \frac{t_w - t_b}{\delta} = \frac{\Delta t}{\delta} \qquad (2)$$

where t_w is the wall temperature, and t_b is the bulk temperature of the flowing fluid.

If Equations (1) and (2) are combined, the resulting equation for heat transfer to a fluid in turbulent motion is

$$dq = \frac{k(\Delta t)}{\delta}\, dA \qquad (3)$$

Because of the difficulty of measuring film thicknesses, this equation is put on an empirical basis by defining a local heat transfer coefficient as k/δ. The result is the familiar equation

$$dq = h\Delta t\, dA \qquad (4)$$

A similar approach to laminar-flow heat transfer is not so easily justified because the temperature of the fluid at a given cross section varies continuously from the tube wall to the axis of flow, and there is no dissimilar region of flow at the boundary which can be used to account for the major part of the resistance to heat transfer. Nevertheless, the practice has been to define heat transfer coefficients for laminar flow exactly as for turbulent flow. In this case the bulk temperature of the fluid does not correspond even approximately to the temperature of any large segment of the fluid, and the Δt, defined as the wall temperature minus the bulk temperature, has no physical significance as a driving force. The bulk temperature of a flowing fluid is defined as the temperature which would be measured if the fluid were run into an adiabatic cup and completely mixed. It is sometimes called the *mixing-cup* temperature.

Experimental values of the local heat transfer coefficient for laminar flow are virtually nonexistent in the literature. Almost all measurements made in the past have been of *mean* values of h for finite lengths of tubing. In the absence of any known relationship among the variables, integration of Equation (4) is carried out to give

$$q = h_m(\Delta t)_m A \qquad (5)$$

The apparent simplicity of this equation is misleading. The main point is that the mean values of h defined by this equation have no significance except in relation to the kind of mean used for $(\Delta t)_m$. Since no method has been available for determining the *proper* mean to be used, the universal practice has been to define $(\Delta t)_m$ *arbitrarily* as the arithmetic mean of the values at the ends of the section considered.

Another important point to be kept in mind when the mean values of h reported in the literature are considered is that the finite lengths of tubing over which they apply invariably include the entrance length, i.e., the length of tube beyond the entrance which is required for the development of complete velocity and temperature profiles in the fluid. This length is usually small for turbulent flow and is relatively unimportant. In laminar flow, however, it has a large influence. The temperature changes which occur as a fluid enters and moves along a heated tube may be considered here. At the entrance the fluid temperature is uniform over the cross section. As the fluid passes into the heated tube, temperature changes occur first in the fluid adjacent to the wall. Gradually the effect of heating progresses to the center of the tube, and as the fluid moves away from the entrance the zone of temperature uniformity narrows until it has only a differential diameter at the center of the tube. The length required for the complete temperature profile to be developed is known as the *thermal entrance length*. For the

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case of fluids having properties independent of temperature and flowing in a circular tube with constant heat flux at the wall, Levy (*3*) has derived an equation giving this length as

$$(L)_t = \frac{(112)(D)(N_{Pr})}{(N_{Re})(f/2)^{0.5}} \qquad (6)$$

where N_{Pr} is Prandtl number, N_{Re} is Reynolds number, D is tube diameter, and f is the Fanning friction factor.

The mechanism of heat transfer to a fluid in laminar flow is conduction. The basic equation is Equation (1), and this fact should not be lost sight of when empirical equations involving arbitrary heat transfer coefficients are substituted for it. At a given cross section the temperature profile in a fluid being heated or cooled in laminar flow depends on the mode of heating or cooling and not alone on the bulk properties of the flowing fluid. Thus the value of h at a given location along a heated tube with a constant wall temperature throughout its length is different from the local value of h measured in another tube at exactly the same conditions of fluid bulk temperature and tube-wall temperature but with a constant heat flux at the wall rather than a constant wall temperature.

Many attempts have been made to solve the problem of laminar-flow heat transfer analytically, but no general solution has been achieved, and the equations which have been developed are based on assumptions which make them unreliable for general application. The basic work has been described by Drew (*4*); however, analytical solutions have served the purpose of suggesting functional relationships among the variables involved. Thus the Graetz solution (*1*) indicates that the mean Nusselt number should be a function of the mean Reynolds number, the mean Prandtl number, and the ratio of the tube diameter to the heated length as follows:

$$(N_{Nu})_m = \phi[(N_{Re})_m(N_{Pr})_m(D/L)]$$

The case considered is for a fluid with properties independent of temperature, for a constant tube-wall temperature, and for a tube length which includes the entrance length.

Sieder and Tate (*5*) were indeed able to correlate the data for liquids flowing in laminar flow inside tubes of constant wall temperature by an equation of this form. In order to bring the data for heating and cooling runs together, they found it necessary to include a viscosity-ratio term, $(\mu/\mu_w)_m$, where μ is the bulk viscosity of the fluid and μ_w is the viscosity of the fluid at the temperature of the tube wall. Their final correlation was

$$(N_{Nu})_m = 1.62[(N_{Re})_m(N_{Pr})_m(D/L)]^{1/3} \cdot \left(\frac{\mu}{\mu_w}\right)_m^{0.14} \qquad (7)$$

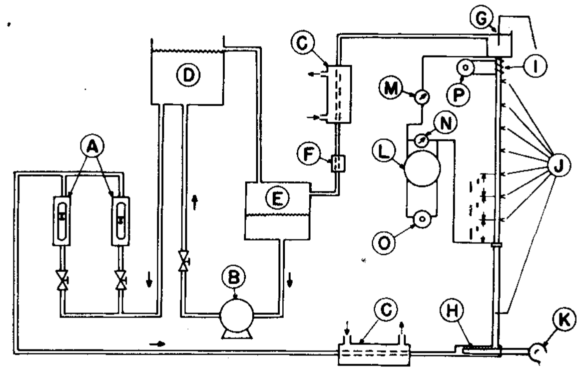

Fig. 1. Experimental apparatus: *A*, rotameters; *B*, pump; *C*, coolers; *D*, constant-head tank; *E*, storage tank; *F*, filter; *G*, mixing cup; *H*, *I*, heaters; *J*, thermocouples; *K*, *O*, *P*, variable transformers; *L*, welding transformer; *M*, ammeter; *N*, voltmeter.

This equation has the advantage of being dimensionless. It is similar in form to equations used for the turbulent region and has been used in the preparation of a single graph which represents heat transfer data for all three regions of flow. In the laminar region separate lines are shown for different values of L/D. In the turbulent region a single line represents all the data. In the transition region lines are sketched in to connect the separate lines for various values of L/D in the laminar region with the single line of the turbulent region. This graph has been widely published in the chemical engineering literature. In using it, one should keep in mind that the values of the heat transfer coefficient which it correlates in the laminar region are average values based on arithmetic-mean temperature differences and that they are valid only when the wall temperature is constant and when L includes the entrance length. The values of h correlated in the turbulent region are essentially local values, subject to none of the foregoing restrictions. The significance of the values of h predicted for the transition region by this correlation is open to speculation, for the lines in this region merely connect a correlation for mean values of h subject to severe restriction to a line for local values subject to none.

An alternative approach to the problem of laminar-flow heat transfer would be a return to the basic equation for heat conduction, Equation (1). Since all heat transferred to or from the fluid must pass through a differential lamina of fluid adjacent to the wall, the temperature gradient in the fluid at the wall may be used in Equation (1) together with the cylindrical surface area of the tube to give the heat transfer rate to the fluid. This equation for the case of heating becomes

$$dq = k_w\left(\frac{dt}{dr}\right)_w dA \qquad (8)$$

where the subscript w denotes values adjacent to the tube wall. Thus the local heat transfer rate at any location along the tube can be calculated from the thermal conductivity of the fluid and the radial temperature gradient in the fluid adjacent to the wall. Laminar-flow heat transfer should be considered in relation to a temperature gradient of physical significance rather than with respect to an abstract coefficient and a meaningless Δt. The experimental problem is that of measuring and correlating the radial temperature gradients at the wall.

An energy balance over a differential length of tube dx in which the bulk temperature of the fluid increases by dt_b gives

$$dq = mc\, dt_b \qquad (9)$$

where m is mass flow rate and c is specific heat. Also

$$dA = \pi D\, dx \qquad (10)$$

Equations (8), (9), and (10) combine to give

$$\left(\frac{dt}{dr}\right)_w = \frac{mc}{\pi D k_w}\left(\frac{dt_b}{dx}\right) \qquad (11)$$

Experimentally, the problem is to determine values of the bulk temperature t_b at various locations along the tube so that dt_b/dx may be evaluated. The technique developed for this purpose involves a series of measurements in which successively shorter heat transfer sections are used. The conditions in each shorter length of tube exactly match those in an equal length from the entrance of the preceding longer tube. The measurement of the bulk temperatures of the fluid flowing from these various lengths amounts to the measurement of the bulk temperatures of the fluid at various points in the original full-length tube. From these measurements, values of the derivative dt_b/dx are readily calculated for use in Equation (11).

EXPERIMENTAL

The general technique just described is applicable in various types of experiments. The particular measurements made in this

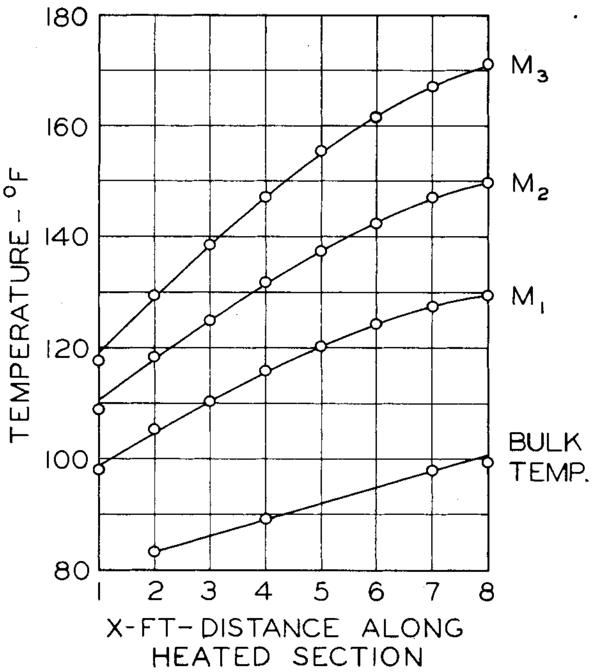

Fig. 2. Experimental data: runs made with 50% aqueous ethylene glycol in the 0.305-in. tube; M_1 = 0.0148 lb./sec., M_2 = 0.0288 lb./sec., M_3 = 0.0428 lb./sec.

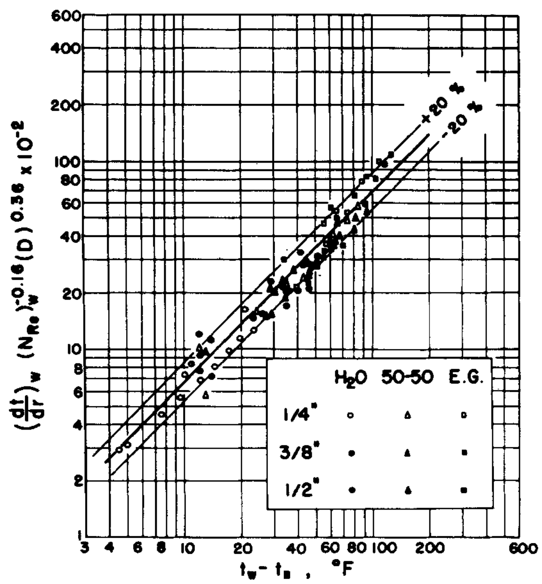

Fig. 3. Correlation of results: 1/4, 3/8, and 1/2 in. are the nominal tube sizes referring respectively to tubes having internal diameters of 0.224, 0.305, and 0.432 in.

work to test the utility of the method involved the heating of three liquids flowing upward in vertical tubes with constant heat flux at the wall. The three liquids used were water, ethylene glycol, and a 50%-by-weight aqueous ethylene glycol solution. The properties of water are readily available, and those of ethylene glycol and its aqueous solutions are given by Cragoe (*6*). Three tube sizes were used: 0.224 in. I.D., 0.305 in. I.D., and 0.432 in. I.D. All were made of austenitic stainless steel. The equipment is shown schematically in Figure 1. The test section of stainless steel tubing, shown at the right, was initially 12 ft. long. Thermocouples were attached to the tube at 1-ft. intervals, and the entire tube was enclosed in a Vermiculite-filled box. An adiabatic mixing cup was fastened to the top of the test section, and a thermocouple was used to measure the bulk temperature of the fluid after complete mixing. The stainless steel tube was heated by passing a high current at low voltage directly through the tube wall, thus giving a constant rate of heat input with length. A welding transformer was used to supply power. Current was fed to the tube through clamps, one attached to the top of the tube and the other at a point 8 ft. lower. The initial 4-ft. length of tube was used as a calming section. Heat losses from the upper clamp were compensated by winding it with electrical heating wire. Heat was supplied at a rate such that no temperature gradient existed in the clamp.

The fluid to the test section, supplied from a constant-head tank, was metered in calibrated rotameters and controlled by regulating valves. Before entering the test section, it passed through a cooler and a heater so that its initial temperature might be adjusted to any desired value. The fluid from the mixing cup was cooled and returned to the system. Runs were made with the full length of tube for each fluid at three mass-flow rates and for three bulk-temperature increases between the tube entrance and the mixing cup. These initial runs were made primarily to establish the wall-temperature profiles along the tube.

The tube was then cut 1 ft. from the top, and the mixing cup was remounted. All runs were repeated with the new tube length. In each case the original entrance conditions of the fluid were duplicated, and the power supplied to the tube and top clamp was adjusted to give wall-temperature profiles along the remaining tube identical with those originally observed. Thus the temperature histories of the fluid in the tube up to the point where the tube was cut were made to duplicate those for the initial runs up to the same point. For each cutting of the tube this procedure was repeated. For the case of constant heat input along the tube length, only one additional cut was needed because intermediate bulk temperatures could be calculated by an energy balance. In the present work, therefore, the only other cut made in most instances was at a point 5 ft. below the first one. In a few cases another cut was made to provide a check on the calculations for intermediate points.

RESULTS

As an example of the data, Figure 2 shows the results of one set of runs taken for the 0.305-in. I.D. tube with 50% aqueous ethylene glycol as the fluid. The three upper curves represent wall-temperature profiles for three different mass-flow rates. The bottom line represents the bulk temperature of the fluid, which was made the same for all three flow rates by adjusting the power input. The wall temperatures are those at the inside of the tube and are average values. They were calculated from the values measured at the outside of the tube by Equation (12):

$$t_{in} = t_{out} + \left(\frac{E}{L}\right)^2 \frac{(r_2^2 - r_1^2) - 2r_2^2 \ln (r_2/r_1)}{4k_t \rho_t} \qquad (12)$$

where E/L is the voltage drop per unit length of tube and r_2 and r_1 are the outside and inside radii of the tube. This equation is readily derived from an energy balance if k_t and ρ_t are assumed constant and if heat losses from the outside of the tube are assumed negligible. The bulk-temperature line shown in Figure 2 is based on the measured values at $x = 2$ and $x = 7$ ft. only. If constant heat input is assumed, the heating rate per foot of length may be determined from two bulk-temperature measurements. Temperatures at other points are then easily calculated if the specific heat is known as a function of temperature. The complete data are given by Gross (*2*).

The slopes of the bulk-temperature curves taken from data such as those shown in Figure 2 were used with Equation (11) to calculate values of $(dt/dr)_w$. Eighty-five separate sets of runs were made under steady state conditions. With this amount of data it was not thought necessary to use more than one value of $(dt/dr)_w$ from each set of runs for purposes of correlation. Hence the correlation of results is based on data for the 5-ft. point in the tubes. This point was far enough from the entrance to lie beyond the thermal entrance length in most cases and far enough from the end not to be influenced by possible end effects. A purely empirical correlation of the values of $(dt/dr)_w$ was attempted. It was postulated that the radial temperature gradient in the fluid at the wall should be a function of N_{Re}, N_{Pr}, D, μ/μ_w, and Δt. Therefore it was assumed that

$$\left(\frac{dt}{dr}\right)_w = B(N_{Re})^a(N_{Pr})^b(D)^c(\mu/\mu_w)^d(\Delta t)^e$$

where B, a, b, c, d, and e are constants. On this basis the data were correlated by a multiple linear-regression analysis based on the method of least squares. The computations were carried out by an Electrodata digital computer. The correlation was successful, but suggested a change in the postulated functional relationship. The following *dimensional* equation was the final result:

$$\left(\frac{dt}{dr}\right)_w = 68(N_{Re})_w^{0.16}(D)^{-0.36}(\Delta t) \quad (13)$$

where

$(dt/dr)_w$ = temperature gradient in the fluid at the wall, °F./ft.

$(N_{Re})_w$ = Reynolds number of the fluid evaluated at the wall temperature

D = inside tube diameter, in.

Δt = $t_w - t_b$, °F.

The correlation is shown graphically in Figure 3. It will be seen that most of the points lie within 20% of the line representing Equation (13).

If a local heat transfer coefficient is defined in the usual way, the correlation may be expressed in an alternative form. By Equations (4) and (8),

$$h = \frac{k_w(dt/dr)_w}{\Delta t}$$

It follows from Equation (13) that

$$h = 68k_w(N_{Re})_w^{0.16}(D)^{-0.36} \quad (14)$$

where h is a local value of the heat transfer coefficient in B.t.u./(hr.)(sq. ft.)(°F.) and k_w is the thermal conductivity of the fluid at the wall temperature in B.t.u./(hr.)(sq. ft.)(°F./ft.).

DISCUSSION

If f in Equation (6) is replaced by $16/N_{Re}$ for laminar flow, the result is

$$(L)_t = (39.5)(D)(N_{Pr})(N_{Re})^{-0.5} \quad (15)$$

Calculations of the thermal entrance lengths by Equation (15) for the runs made showed the 5-ft. point to be within the entrance region for five of the ethylene glycol runs. These points failed to fit the correlation and were discarded, but all other points are included. On the basis of excellent energy balances, the data are thought to be accurate to within 10%.

The correlation of results applies specifically to liquids flowing upward in vertical tubes under conditions of constant heat flux at the wall and gives local values for the case of fully developed temperature profiles. It is not suggested that this correlation should be valid under any other circumstances. It was thought that liquids heated in upward flow would be less susceptible to the effects of free convection than in any other arrangement. The heating of liquids in horizontal tubes or in downward flow in vertical tubes should follow the present correlation provided that free-convection effects are negligible.

It has not been possible to compare the present results with those of any previous work. No data are available in the literature for local heat transfer rates to liquids in laminar flow. Furthermore, the results of this work are applicable only in the region of fully developed temperature and velocity profiles. All mean values of h reported in the literature include the entrance region. Since the results of this work do not apply to this region, it is not possible to integrate to get mean values of h comparable with those given in the literature. It might be added that most data available from previous work are for the case of constant wall temperatures, whereas the present work was concerned with the case of constant heat flux at the wall.

The results presented are thus seen to be of limited practical utility. However, it is hoped that the presentation of this experimental method will stimulate efforts toward further research on laminar-flow heat transfer. The method is by no means limited to the case of constant heat flux at the wall, and it should not be difficult to devise experiments for the study of other cases. An interrelation among the local heat transfer rates for different modes of heating and cooling may be found, and it is possible that a single correlation of such rates might be made through the introduction of other factors. The entrance region might well be treated separately from the region of fully developed profiles. A detailed study of entrance effects is also possible by this method. In addition, point values of heat transfer rates in the transition region might be investigated. However, this may involve added experimental difficulties. In the course of the present work the region of transition flow was inadvertently entered several times, and it was noted that the temperatures fluctuated wildly, a result that would be expected because of the unstable nature of transition flow. Certainly, further work on these problems is needed.

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NOTATION

A = area
c = specific heat
D = tube diameter
E = voltage drop
f = Fanning friction factor
h = local heat transfer coefficient
k = thermal conductivity of fluid
k_t = thermal conductivity of the tube wall
L = tube length
L_t = thermal entrance length
m = mass flow rate
N_{Nu} = Nusselt number, hD/k
N_{Pr} = Prandtl number, $c\mu/k$
N_{Re} = Reynolds number, $Dv\rho/\mu$
q = rate of heat transfer
r = tube radius
s = distance in the direction of heat flow
t = temperature
t_w = tube-wall temperature
t_b = bulk temperature of fluid
v = velocity of fluid
x = distance along tube
δ = film thickness
ρ = density of fluid
ρ_t = electrical resistivity of tube
μ = viscosity of fluid at bulk temperature
μ_w = viscosity of fluid at wall temperature

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